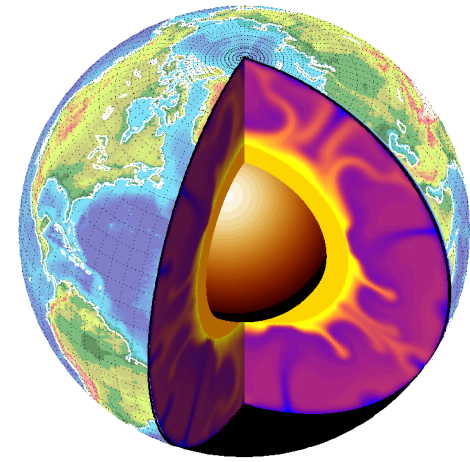


Nuclear processes in the Earth's interior



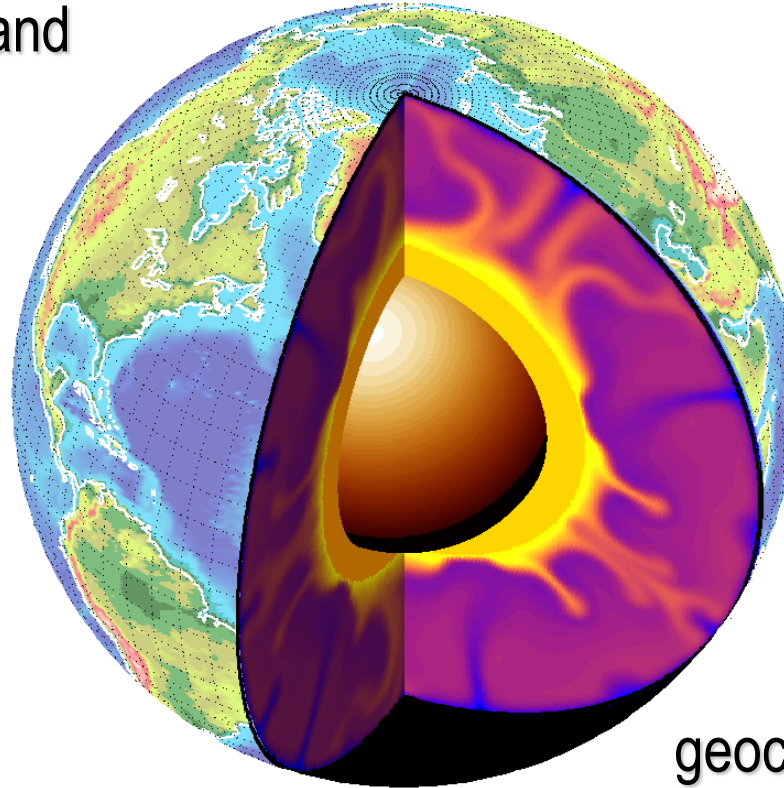
1. Heat release from the Earth
2. Energy sources inside the Earth
3. The radiogenic component of the Earth's energy
4. The decay chains of U, Th and ^{40}K
5. Models of the Earth interior
6. Estimates of Geo-neutrino fluxes

Open questions about natural radioactivity in the Earth

1 - What is the radiogenic contribution (from U, Th and ^{40}K) to terrestrial heat production?

2 - How much U and Th in the crust?

3 - How much U and Th in the mantle?

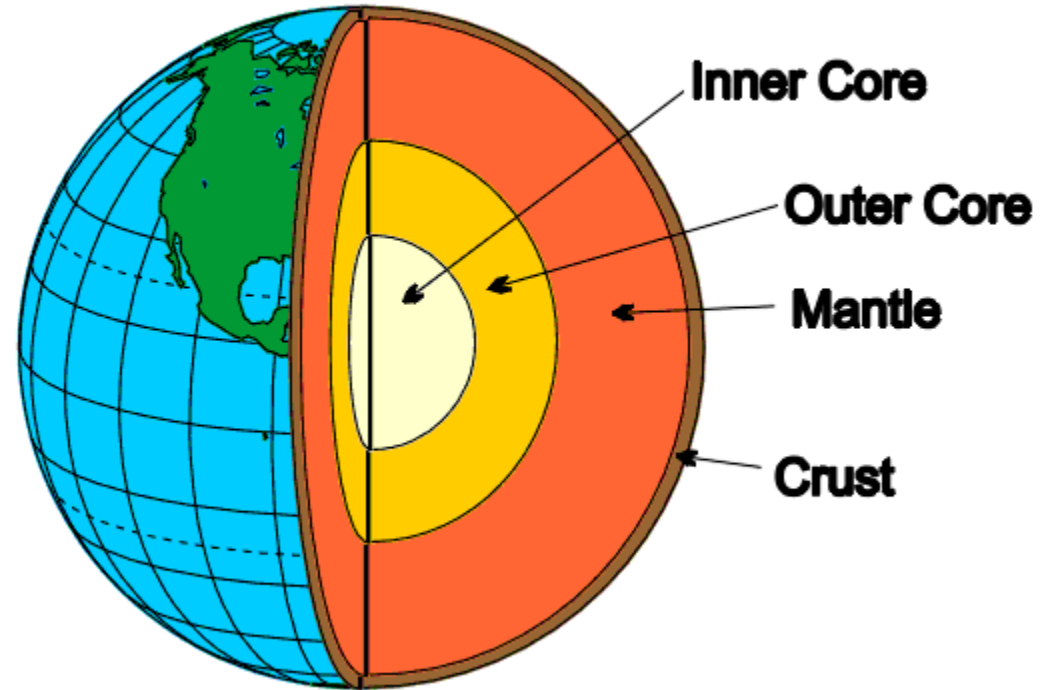


4 - What is hidden in the Earth's core? (geo-reactor, ^{40}K , ...)

5 - Is the standard geochemical model (BSE) consistent with geo-neutrino data?

Fundamentals about Earth's structure

1. The crust is solid rock (silicates) varying in thickness (on the average 30 km below continents, 10 km below oceans), with density of about 3 Ton/m^3
2. The Mantle (down to 2890 km) is molten rocks rich in magnesium and silicon, with density of about 5 Ton/m^3
3. The outer core (2890- 5150 km) is liquid, containing mainly iron with some nickel.
4. The inner core (5150- 6360 km) is solid, containing iron and some nickel, with very high pressure and a temperature over 3000°



Mean radius $6.4 \cdot 10^3 \text{ km}$

Mass $6.0 \cdot 10^{24} \text{ kg}$

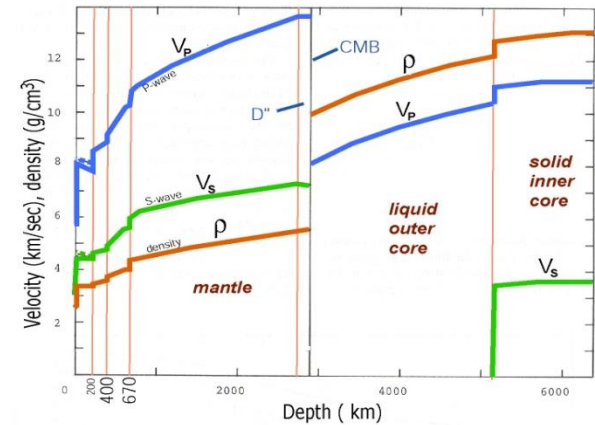
Mean density 5.5 g/cm^3

Heat release: $\approx 40 \text{ TW}$



Probes of the Earth's interior

- Deepest hole is about 12 km
- Samples from the crust (and the upper portion of mantle) are available for geochemical analysis.
- Seismology reconstructs density profile (not composition) throughout all Earth.



Geo-neutrinos: a new probe of Earth's interior

- ✓ They escape freely and instantaneously from Earth's interior.
- ✓ They bring to Earth's surface information about the chemical composition of the whole planet.



Heat flux from the Earth

- Earth radiates back the energy which it receives from the Sun adding to it its own (small) contribution*.
- There is a tiny flux of heat coming from the Earth.

$$\Phi \approx 80 \text{ mW/m}^2$$

- It is much smaller than the flux from the Sun:

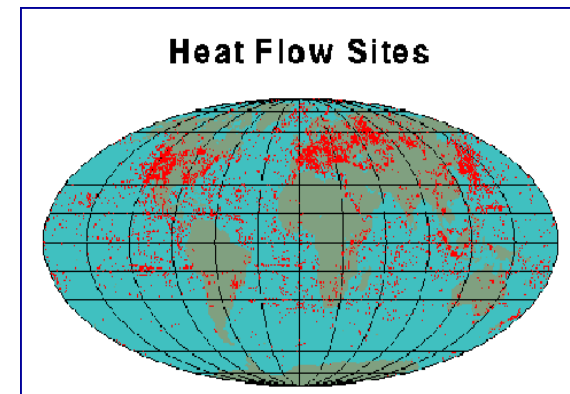
$$\Phi_{\text{Sun}} = 1.4 \text{ kW/m}^2$$

- It is much larger than that from cosmic rays**:

$$\Phi_{\text{cos}} \approx 10^{-8} \text{ W/m}^2$$

*) **Exercise 1:** Compute the equilibrium temperature of the Earth, if it radiates as a black body the energy received from the Sun, corresponding to 70% of the solar constant

) **Exercise 2: estimate Φ_{cos} assuming that at sea level one has 1 muon/cm²/minute, with a typical energy of 1 GeV



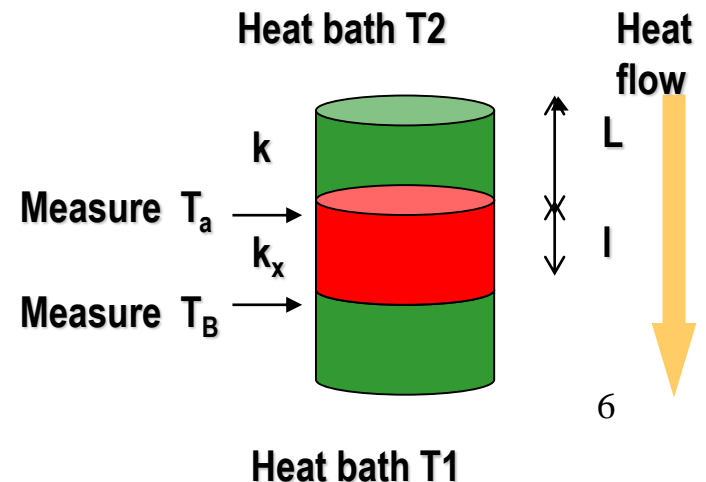
The Fourier law for heat conduction

- Heat conduction satisfies the Fourier law. If temperature T varies along (say) the z -axis, heat flux Φ is proportional to the temperature gradient, $\Phi = k \, dT/dz$, where k is the thermal conductivity of the material.
- This is clearly the equivalent of the electrical conductivity law $\mathbf{J} = \sigma \mathbf{E} = \sigma \nabla V$
- The thermal conductivity coefficient has dimensions
 $[k] = [\text{Energy}]/L/t / (^\circ\text{K})$ and is measured in $\text{W}/\text{m}/(^\circ\text{K})$
- The thermal conductivity of a rock is of order unity in these units.
- Thermal conductivity of rock sample can be measured by the “divided-bar” method, where one determines the ratio of the unknown conductivity (k_x) to that of a well known specimen (k). The method is much the same as that of measuring an electric resistance, through comparison to a standard resistance, see figure:



$$\Phi = k (t_2 - T_a) / L = k_x (t_b - T_a) / l$$

$$k_x = k(l/L) (t_2 - T_a) / (t_b - T_a)$$



Measurements of heat flux

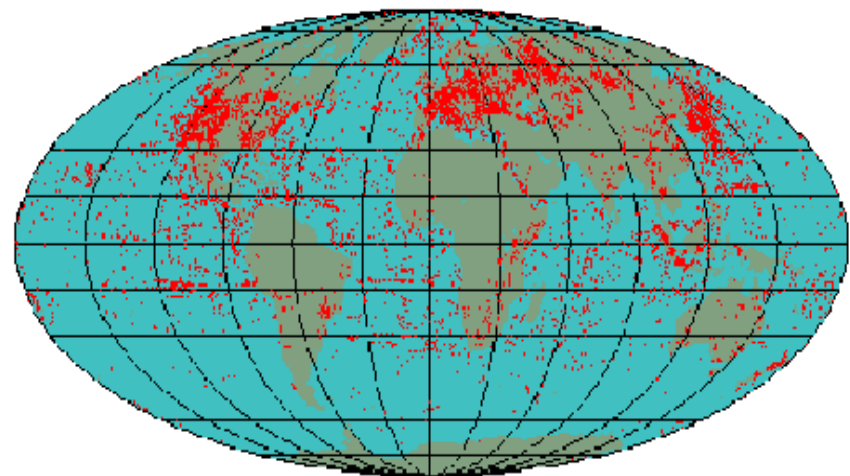
- Measure temperature at different depths ($L \approx 1\text{km}$)
- Get samples of rocks in order to determine their conductivity k
- Derive heat flow from dT/dz and rock conductivity k :

$$\Phi = k \, dT/dz$$

- Measurements in the oceans are more uncertain, due to heat removal from water in the soft sediments.

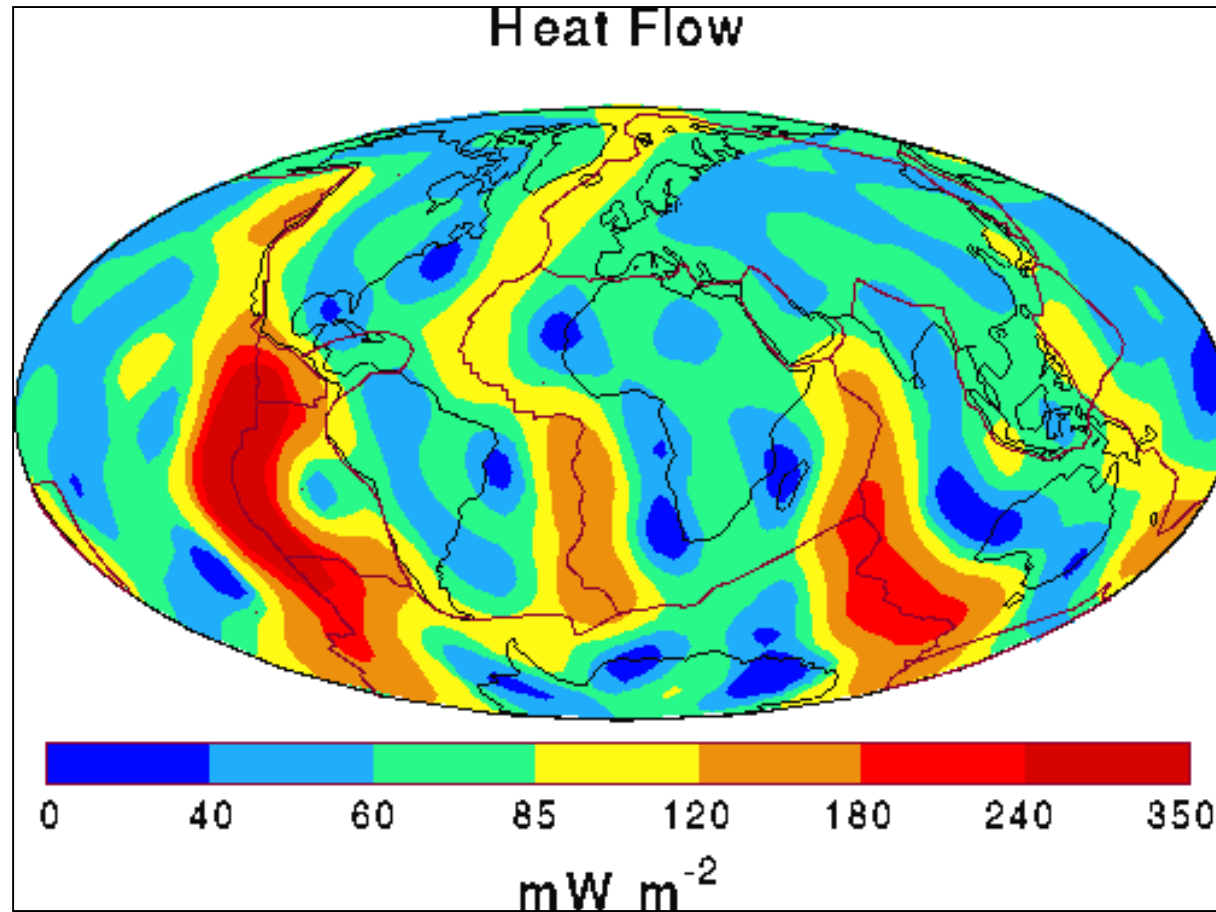


Heat Flow Sites



Reconstruction of the heat flow

- Data embodied in this map are more than 24,000 field measurements in both continental and oceanic terrains.
- Observations of the oceanic heat flux have been corrected for heat loss by hydrothermal circulation through the oceanic crust.



- The map is a representation of the heat flow

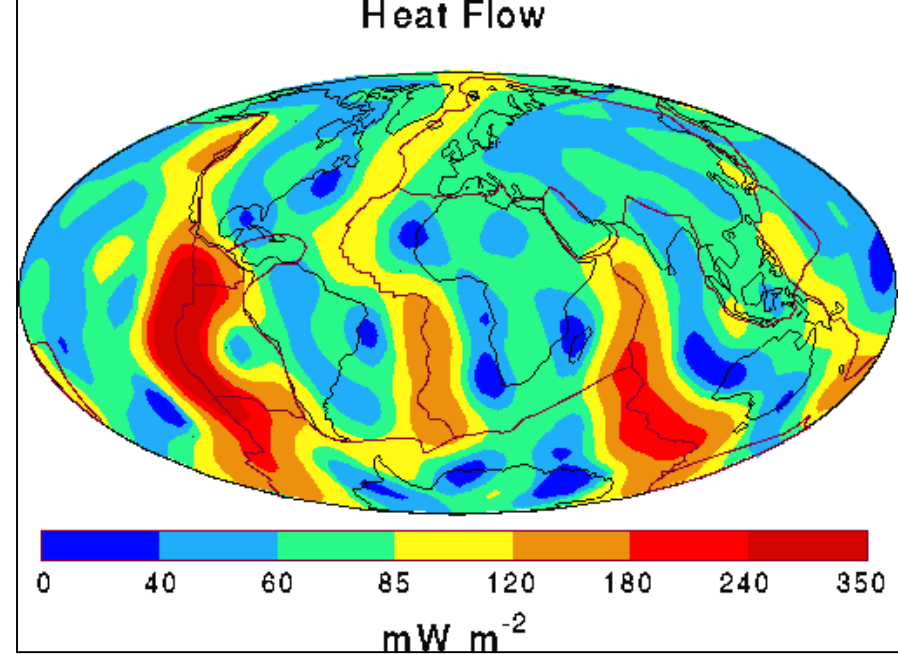
Heat flow from the Earth

- By integrating the flux one can get the total flow (with uncertainty of maybe 20 %).

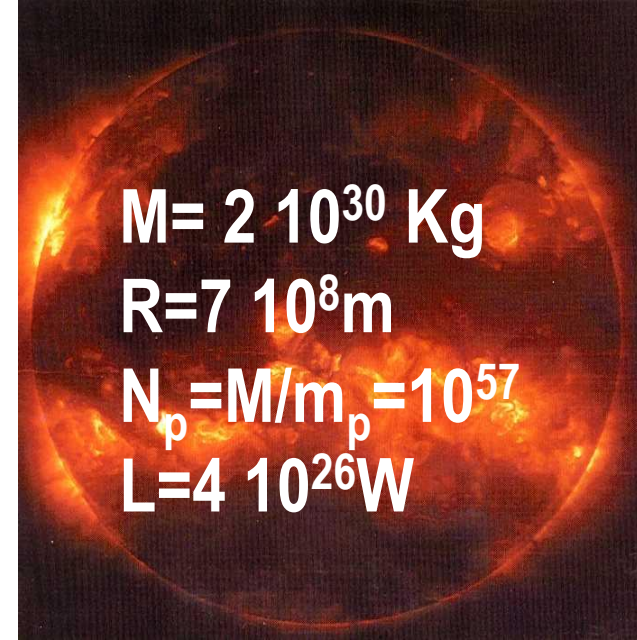
- There is a **huge** flow of heat from the Earth:

$$H_E = 4 \cdot 10^{13} \text{ W} = 40 \text{ TW}$$

- It is the equivalent of 10^4 nuclear power plants.
- It is comparable to the human power production.
- **Where does it come from?**



The Sun energy inventory



- It is easy to understand the dominant contribution to the solar energy production.
- The present luminosity $L = 4 \cdot 10^{26} \text{ W}$ can be sustained by an energy source U for a time $t = U/L$:
 - a) chemistry: $U \approx (0.1 \text{ eV}) N_p = 1 \text{ eV} \cdot 10^{56} = 2 \cdot 10^{37} \text{ J}$ $\rightarrow t_{\text{ch}} = 2 \cdot 10^3 \text{ y}$
 - b) gravitation $U \approx GM^2/R = 4 \cdot 10^{41} \text{ J}$ $\rightarrow t_{\text{gr}} = 3 \cdot 10^7 \text{ y}$
 - c) nuclear $U \approx (1 \text{ MeV}) N_p = 2 \cdot 10^{54} \text{ J}$ $\rightarrow t_{\text{gr}} = 2 \cdot 10^{10} \text{ y}$
- Thus only nuclear energy is important for sustaining the Solar luminosity over the sun age, $t = 4.5 \cdot 10^9 \text{ y}$.

The Earth energy inventory

It is **not at all** easy to understand the dominant contribution to the **Earth** energy production.

• The present heat flow $H_E = 40 \text{ TW}$ can be sustained by an energy source U for a time $t = U/H_E$:

• a) "chemistry"*) : $U \approx (0.1 \text{ eV}) N_p = 6 \cdot 10^{31} \text{ J}$ $\rightarrow t_{ch} = 5 \cdot 10^{10} \text{ y}$

• b) gravitation $U \approx GM^2/R = 4 \cdot 10^{32} \text{ J}$ $\rightarrow t_{gr} = 3 \cdot 10^{11} \text{ y}$

• c) nuclear **) $U \approx (1 \text{ MeV}) N_{p,rad} = 6 \cdot 10^{30} \text{ J}$ $\rightarrow t_{gr} = 5 \cdot 10^9 \text{ y}$

• Thus **all** energy sources seem capable to sustain H_E on geological times.

*) actually it means the solidification (latent) heat, see later

**) the amount of radioactive material is taken as $M_{rad} \approx 10^{-8} M_{Earth}$



The Earth energy inventory: gravitation ...and the rest



- Gravitational energy from building the Earth should have been radiated away very early ($\tau = U / \pi R^2 \sigma T^4 \approx 10^6 \text{ y}$).
- Similarly, “solidification” have produced heat mainly in the early times, and most heat should have been radiated away.
- Gravitation and solidification, however, can work still today:
 - There is a large fraction of the nucleus which is still liquid (as implied by the fact that only longitudinal waves penetrate into it). It may be that in this region solidification occurs even now.
 - With solidification of the core, one forms a higher density material, which sinks to the bottom, releasing gravitational energy
- All this may contribute an amount $L_{\text{LH}} \approx 10^{12} \text{W}$
- Tidal deceleration of the Earth results in dissipation of rotational kinetic energy ($E_{\text{rot}} \approx 2 \cdot 10^{29} \text{ J}$) of similar order of magnitude.

The nuclear contribution

- It is easy to understand that radioactivity can produce heat now at a rate comparable with the observed $H_E=40$ TW.

- Consider the chain : $^{238}\text{U} \rightarrow ^{206}\text{Pb} + \dots$ energy

- The released energy is $\Delta \approx 50\text{MeV}$, the lifetime $\tau \approx 8$ Gy and mass $m_u \approx 200m_p$.

- The energy produced per unit mass and unit time is:

$$\varepsilon = \Delta / (\tau m_u) \approx 1 \text{ erg/g/s}$$

- A Uranium mass $M_U \approx 10^{-8} M_{\text{Earth}}$ gives:

$$H_U = \varepsilon M \approx 6 \text{ TW}$$

- In addition, there are other natural radioactive elements (Th and ^{40}K) capable of providing similar amounts of energy.

A first summary

J Verhoogen, in *Energetics of Earth* (1980) N.A.S.:

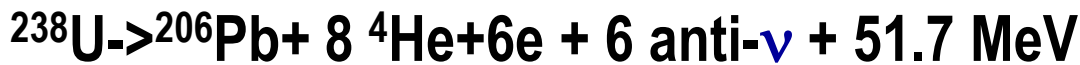
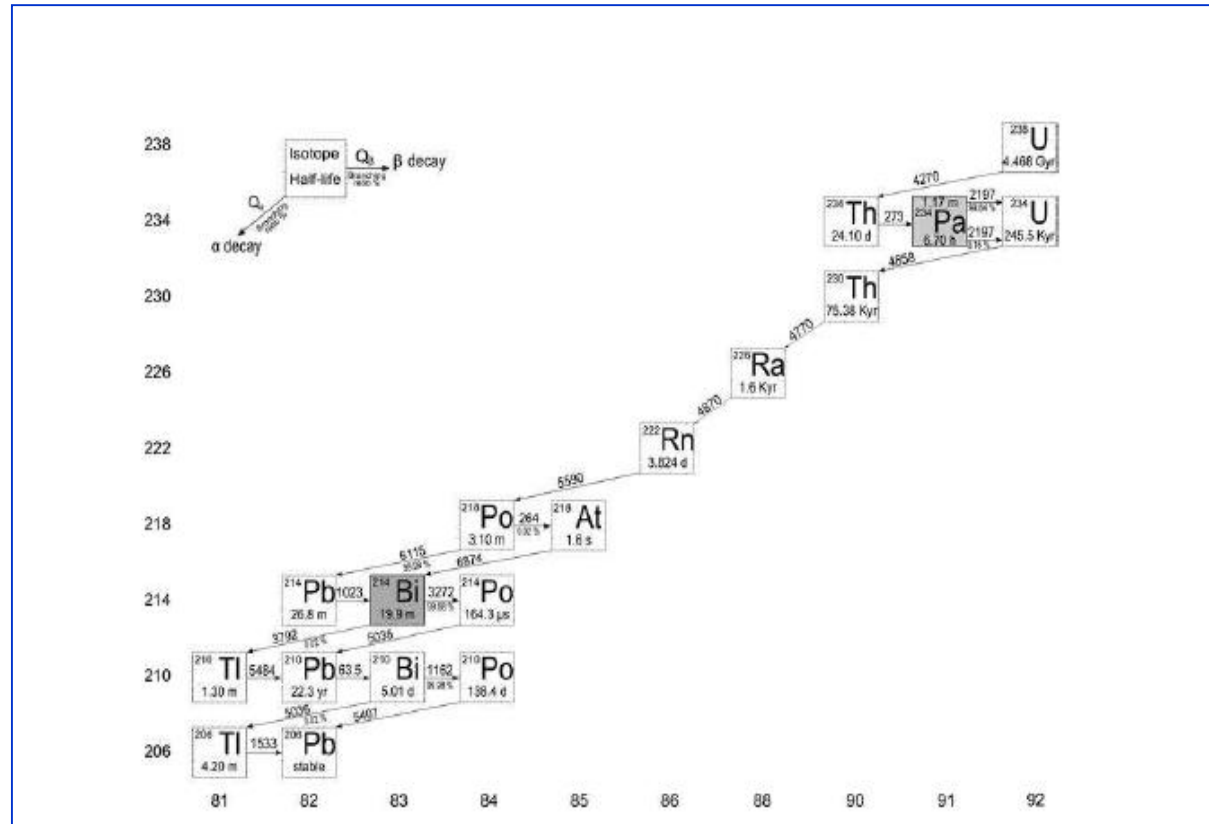
- “What emerges from this morass of fragmentary and uncertain data is that **radioactivity itself** could possibly account for at least 60 per cent if not 100 per cent of the Earth’s heat output”.
- “If one adds the greater rate of radiogenic heat production in the past, possible release of gravitational energy (original heat, separation of the core...) tidal friction ... and possible meteoritic impact ... the total supply of energy may seem embarrassingly large...”
- “Not only must we know where the heat sources are, we must also know when they came into effect and what the **heat transfer** may be.”
- **What is the radiogenic contribution to Earth heat production?**

Long lived radioactive elements in the Earth

- Essentially one has to consider elements which have lifetimes comparable to Earth's age and which have significant abundances.
- This selects three isotopes*:
 ^{238}U , ^{232}Th , ^{40}K
- An important point is that for all these nuclei, **heat is released together with antineutrinos****, in a well fixed ratio, so that antineutrino detection can provide information on the heat which is released.
- *) for completeness, one should add ^{235}U , but this plays a minor role, for our purposes
- **) ^{40}K can emit both neutrinos and antineutrinos, see later

^{238}U decay chain

- ^{238}U Uranium has half life 4.6 Gyr, comparable to the Earth's age.
- Its decay chain includes 8 α -decay and 6 β decays, releasing $Q=51.7$ MeV together with $N= 6$ antineutrinos*



This allows to estimate heat and neutrinos produced per unit mass and time, $\varepsilon(\text{U}) = Q/(\tau m_{\text{U}})$ and $\varepsilon_{\text{antineu}} = N/(\tau m_{\text{U}})$, which gives:

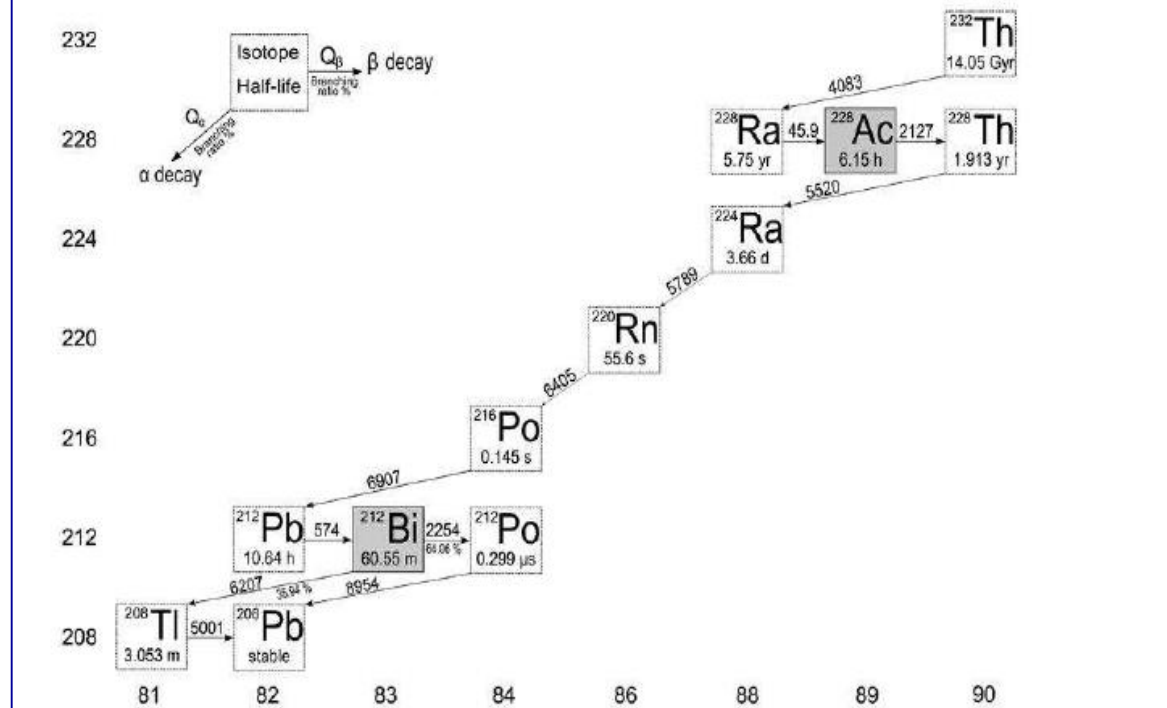
$$\varepsilon(\text{U}) = 0.95 \text{ erg/g/s}$$

$$\varepsilon_{\text{antineu}}(\text{U}) = 7.4\text{E}4 \text{ /g/s}$$

Exercise: if you know that initial (^{238}U) and final nuclei (^{206}Pb) are connected by α and β decays, compute how many are needed by using conservation laws.

^{232}Th decay chain

- ^{232}Th has half life 14 Gyr, comparable to the Earth's age.
- Its decay chain includes 4 α -decay and 4 β decays, releasing $Q=43$ MeV together with $N= 4$ antineutrinos



This allows to estimate heat and neutrinos produced per unit mass and time, $\epsilon = Q/(\tau m_{\text{Th}})$ and $\epsilon_{\text{antinu}} = N/(\tau m_{\text{U}})$, which gives:

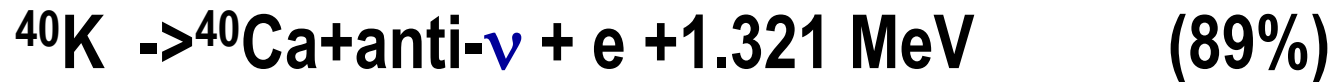
$$\epsilon (\text{Th}) = 0.27\ \text{erg/g/s}$$

$$\epsilon_{\text{antiv}} (\text{Th}) = 1.6\text{E}4\ \text{/g/s}$$

^{40}K decays

- Potassium is the seventh element in the Earth, in order of abundance.
- About 10^{-4} of it is in the form of the long lived, radioactive ^{40}K with two (important) decay modes:

Main article: Isotopes of potassium					
iso	NA	half-life	DM	DE (MeV)	DP
^{39}K	93.26%	^{39}K is stable with 20 neutrons			
^{40}K	0.012%	$1.248(3)\times 10^9$ y	β^-	1.311	^{40}Ca
			ε	1.505	^{40}Ar
			β^+	1.505	^{40}Ar
^{41}K	6.73%	^{41}K is stable with 22 neutrons			



For the present natural isotopic abundance*:

$$\varepsilon(\text{K}) = 3.6 \cdot 10^{-5} \text{ erg/g/s} \quad \varepsilon_{\text{antiv}}(\text{K}) = 27 \text{ /g/s} \quad \varepsilon_{\nu}(\text{K}) = 3.3 \text{ /g/s}$$

***Exercise:** the “Reference Man” contains 140 g of Potassium. Estimate the production rate of neutrinos, positrons and antineutrinos

Summary: long lived radioactive elements produce heat and (anti)neutrinos

•Uranium:



$$\varepsilon\ (\text{U}) = 0.95\ \text{erg/g/s}$$

$$\varepsilon_{\text{antiv}}\ (\text{U}) = 7.4\text{E}4\ \text{/g/s}$$

•Thorium:



$$\varepsilon\ (\text{Th}) = 0.27\ \text{erg/g/s}$$

$$\varepsilon_{\text{antiv}}\ (\text{Th}) = 1.6\text{E}4\ \text{/g/s}$$

•Potassium: about $1.2 \cdot 10^{-4}$ is the long lived ^{40}K :



$$\varepsilon\ (\text{K}) = 3.6 \cdot 10^{-5}\ \text{erg/g/s}$$

$$\varepsilon_{\text{antiv}}\ (\text{K}) = 27\ \text{/g/s}$$

$$\varepsilon_{\nu}\ (\text{K}) = 3.3\ \text{/g/s}$$

(ε correspond to present natural Isotopic abundances)

Equations for Heat and neutrinos

- For each elements there is a well fixed ratio heat/ (anti) neutrinos:

$$H = 9.5 M(U) + 2.7 M(\text{Th}) + 3.6 \cdot 10^{-4} M(K)$$

$$L_{\text{anti-}\nu} = 7.4 M(U) + 1.6 M(\text{Th}) + 27 \cdot 10^{-4} M(K)$$

$$L_{\nu} = 3.3 \cdot 10^{-4} M(K)$$

where suitable units are used:

$$H \text{ [TW]}; \quad M \text{ [} 10^{17} \text{kg]}; \quad L \text{ [} 10^{24} \text{ particles /s]}$$

- Everything is fixed in terms of 3 numbers:

$M(U)$, $M(\text{Th})$ and $M(K)$

Or equivalently $M(U)$, $M(\text{Th})/M(U)$ and $M(K)/M(U)$

What do we know about these numbers?

The simplest chondritic Earth

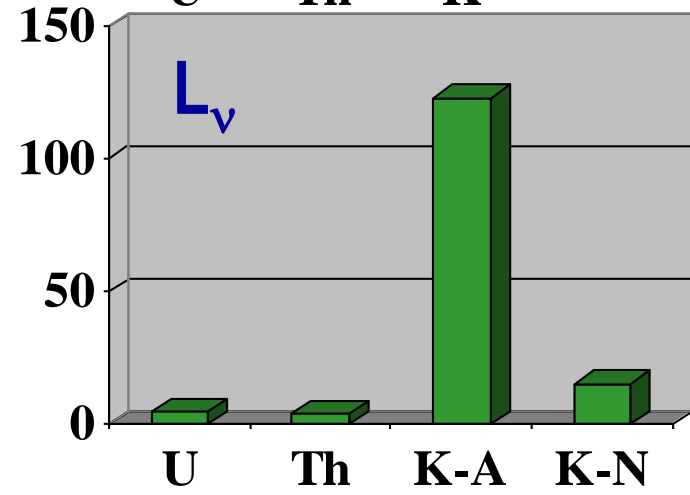
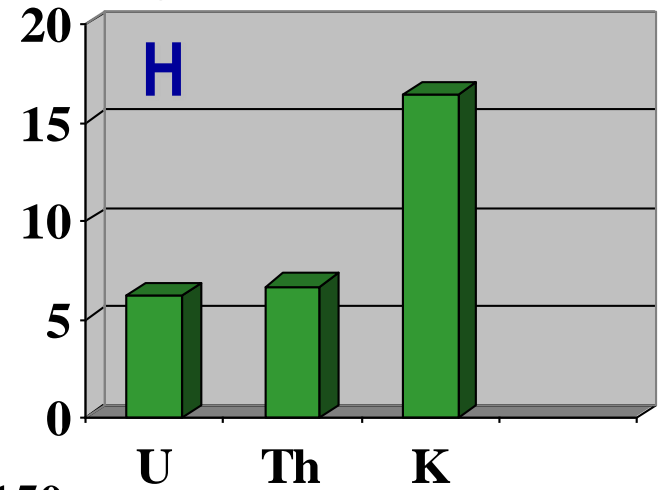
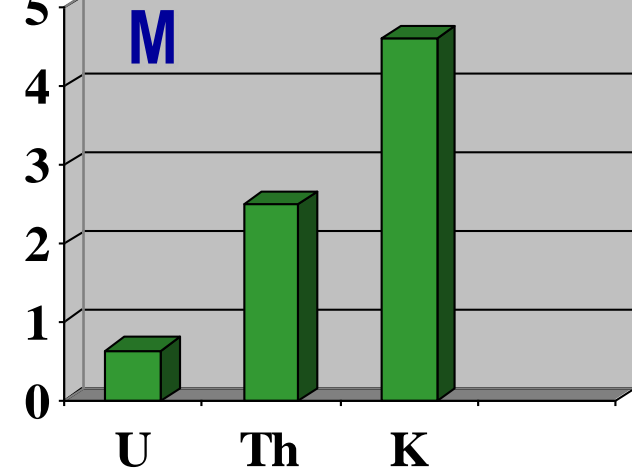
- The oldest objects in the solar system are the Carbonaceous Chondrites (CC), where: $\text{Th}/\text{U} = 3.8$, $\text{K}/\text{U} = 7 \cdot 10^4$ and $\text{U}/\text{Si} = 7.3 \cdot 10^{-8}$, see chapter 1
- Assume that Earth is assembled by the aggregation of CC. (homogeneous accretion).
- Assume that K, Si, U and Th are not lost (or anything is lost in the same way) in the Earth formation*.
- In the Earth $M(\text{Si})/M = 15\%$ and $M_E = 6 \cdot 10^{24}$ kg. This is enough to tell $M(\text{U})$, $M(\text{Th})$, $M(\text{K})$, H and $L_v \dots$

*This is clearly a very rough approximation: volatile elements may be lost in the aggregation problem, see later.

The predictions for the simplest chondritic Earth

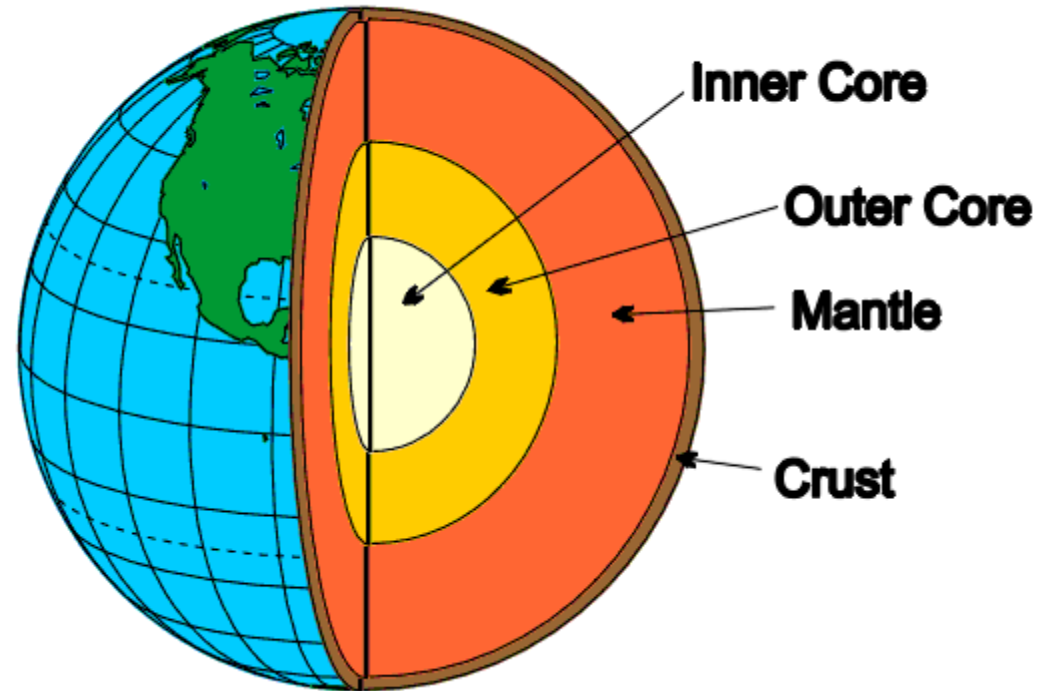
- It immediately accounts for some 30 TW, or (at least) 3/4 of Terrestrial Heat release
- Heat production is dominated by K
- Antineutrino production is also dominated by K

H [TW] **M** [10^{17} kg] **L_v** [10^{24} /s]



Where are U, K and Th ?

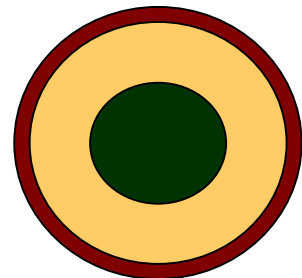
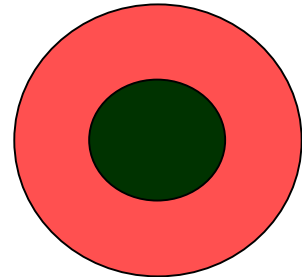
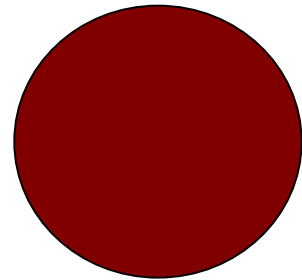
- According to geochemistry U, K and Th are “lithophile”, so they should accumulate in the crust.
- In fact, the crust is the only reservoir which can be easily accessed for sampling and chemical analysis.
- The crust is **estimated** to contain 0.4×10^{17} Kg of U , Th/U is (roughly) consistent with C.C. Value $Th/U \approx 4$, but $K/U \approx 10.000$, a factor seven below CC



- U, Th and K should be absent from the core, which contains iron and “siderophile “elements

The Bulk Silicate Earth model

- Only the upper portion of the mantle can be analyzed, and geochemical arguments are used to infer its composition.
- In the beginning, Earth was homogeneous. Then heavy elements (Iron, nickel and affine elements) precipitated to form the core.
- At that time Earth was divided in two reservoirs, core and “primitive mantle”, or Bulk Silicate Earth (BSE)
- The BSE model predicts $M(U)=0.8$ $Th/U \approx 4$ and $K/U=10.000$
- Crust and present mantle have been separated from the primitive mantle.
- The present mantle is obtained from the BSE by subtracting elements which separated in the crust.
- Thus the BSE predicts that the mantle contains about the same quantities of U, Th and K as the Crust.
- Note the huge concentration of these elements in the crust , e.g. $a_c(U) \approx 10^{-6}$ whereas $a_m(U) \approx 10^{-8}$.



How much potassium?

- Note that $K/U=10,000$, a factor 7 below C. C.
- This raises some problem, since elements as heavy as Potassium should not have escaped from a planet as big as Earth.
- Most reasonable assumption is that K volatilized in the formation of planetesimals from which Earth has accreted (heterogeneous accretion) and is $K/U=10,000$ generally accepted.
- However, it has been suggested that at high pressure Potassium behaves as a metal and thus it could have been buried in the Earth core. The missing potassium might stay in the core, according to some geochemists
- Potassium in the core could provide the energy source of the terrestrial magnetic field.
- However, potassium depletion is also observed in Moon, Venus and Martian rocks.

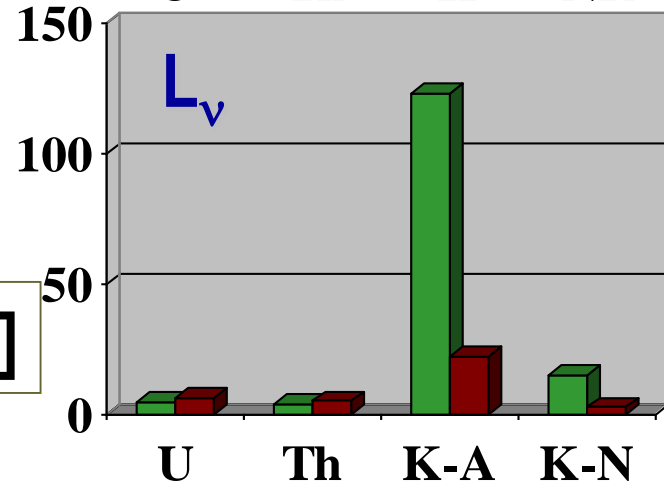
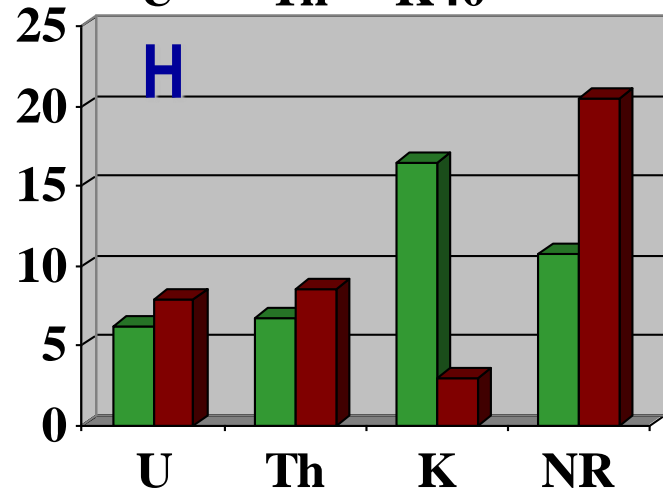
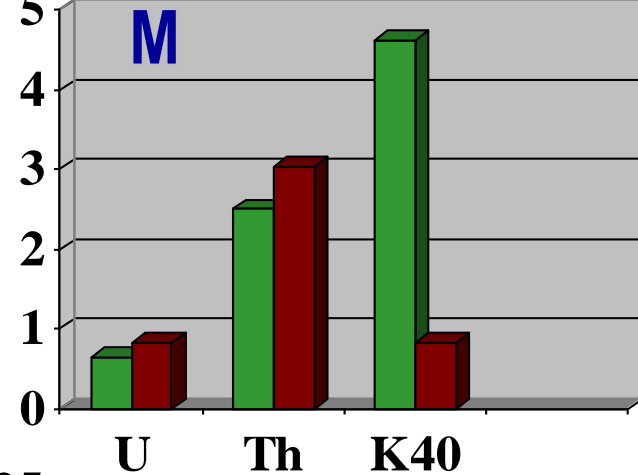
Predictions of the canonical BSE model

- According to BSE,

$$M(U) = 0.8 ; \text{Th}/U = 4; \text{K}/U = 10,000$$

- This accounts for just 1/2 of H_E
- Main radiogenic heat sources are U and Th
- Antineutrino production is any how dominated by K .
- Note that the antineutrino luminosities are:
 - from Potassium: $1.2 \cdot 10^{26}/s$
 - from Uranium: $6 \cdot 10^{24}/s$
 - From Th : $5 \cdot 10^{24}/s$

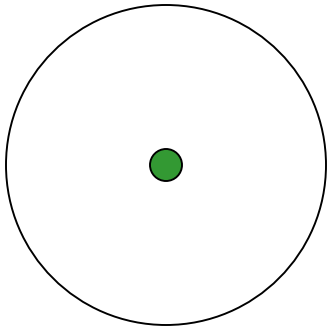
H [TW]	M [10^{17}kg]	L_ν [10^{24} /s]
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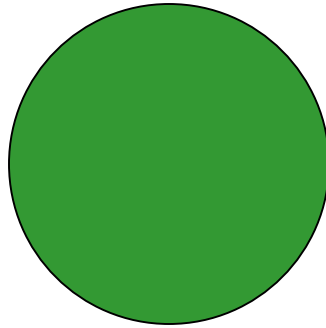
Exercise: build a fully radiogenic model, with $\text{Th}/U = 4; \text{K}/U = 10,000$

From luminosity to fluxes

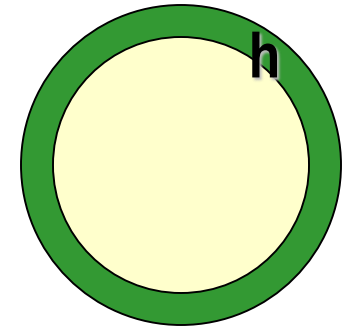
- This depends on the geometry:



$$\Phi_{\nu} = L_{\nu} / 4\pi R^2$$



$$\Phi_{\nu} = 3/2 L_{\nu} / 4\pi R^2$$



$$\Phi_{\nu} = 3.5 L_{\nu} / 4\pi R^2 \quad (h=30\text{Km})$$

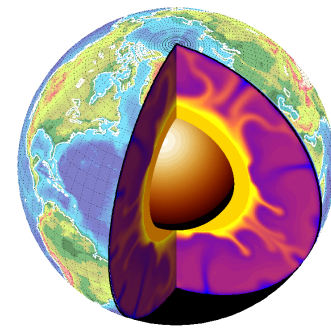
- We shall write $\Phi_{\nu} = G L_{\nu} / 4\pi R^2$ with G an unspecified geometrical factor, of order unity.
- For the BSE model, half of sources are in the mantle and half in the crust, so $G(\text{BSE}) = 2.5$

Exercise: calculate G for the three geometries above

Solar neutrinos and terrestrial antineutrinos

- Note that for a luminosity $10^{24}/s$, one has $\Phi_{\nu}/G = 2 \cdot 10^5 /cm^2/s$
- So neutrino and antineutrinos from the Earth are much less than neutrinos arriving from the Sun, where: $\Phi(pp) \approx 6 \cdot 10^{10} cm^2/s$, $\Phi(Be) \approx 6 \cdot 10^9 cm^2/s$ and $\Phi(B) \approx 6 \cdot 10^6 cm^2/s$.
- However, Earth shines mainly in **antineutrinos**, whereas the Sun emits **neutrinos**.
- This is the reason for concentrating on antineutrino emission
- Antineutrinos from U have a continuous spectrum up to $E_{max} = 3.26 MeV$ and those from Th up to $E_{max} = 2.25 MeV$
- This means they both are above the threshold for the classical detection reaction: **(anti)- $\nu + p \rightarrow n + e^+ - 1.804 MeV$**
- This is not so for antineutrinos from potassium, which have to be detected by some other method (to be invented)

Summary



- Geological observations say that in the crust $M(U) = 0.4 \cdot 10^{17} \text{ Kg}$, ^{40}K is comparable to U and Th is roughly 4 times. Radiogenic heat in the crust accounts for some 10 TW of the 40 TW total observed.
- According to BSE the mantle should contain similar amounts of U, Th and K, thus accounting for another 10 TW
- Questions:
 - Check how much U and T are present in the crust?
 - Measure how much U and Th are in the mantle
 - Detection of antineutrinos from U and Th by means of inverse beta decay on free protons can shed light on the contribution of these elements to terrestrial energy production
 - What is the fate of potassium? How can detect their antineutrino emissions?

The basic question is: What is the true energy budget of the Earth?

Appendix

A fully radiogenic model

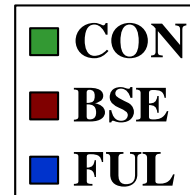
- If one insists that terrestrial heat flow is fully radiogenic, while keeping the terrestrial ratios:

$$\text{Th/U} = 4; \text{K/U} = 10,000$$

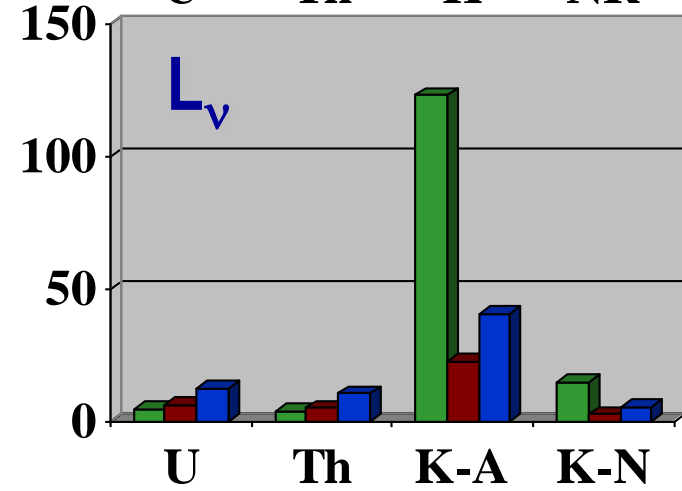
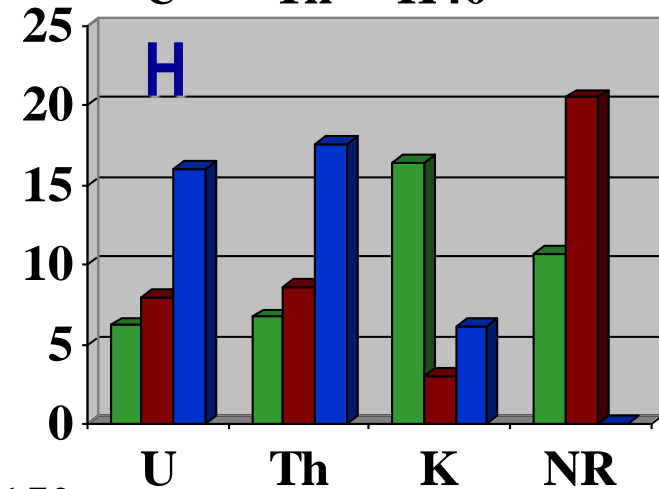
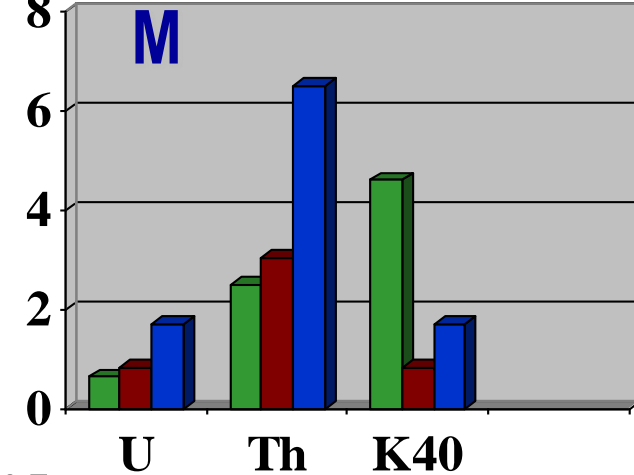
one has to double Uranium mass with respect to BSE:

$$M(\text{U}) = 1.7$$

- Everything is scaled up by a factor of two.



H [TW] **M** [10^{17} kg] **L_v** [10^{24} /s]

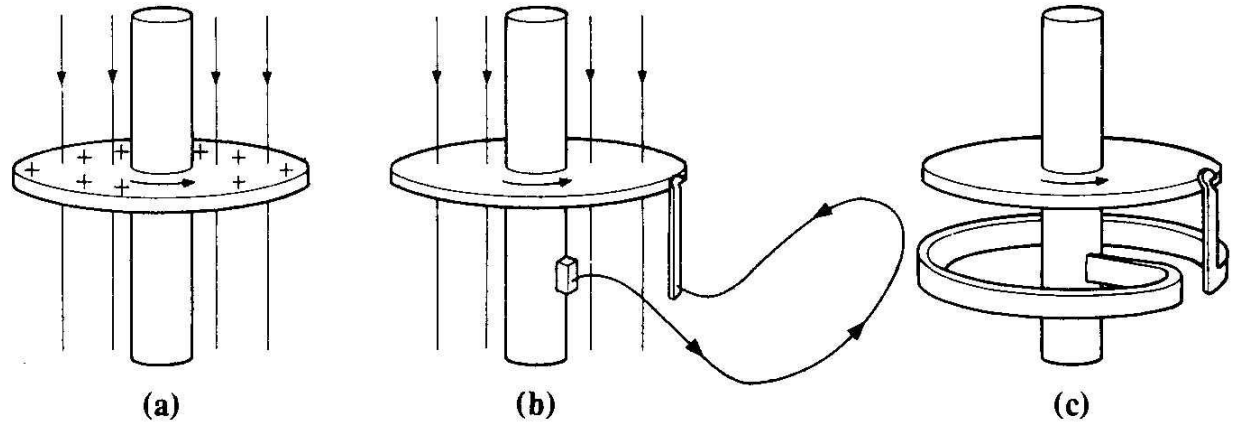


The energy problem of the core

- The Earth magnetic field is presumably generated in the liquid part of the core by means of an (auto excited) dynamo effect. For sustaining the geomagnetic field one needs power in the Earth core.

- One estimates that 10^{11} - 10^{12} Watt of thermal energy are necessary.

- By assuming that Potassium is in the core one finds the energy source.



- Remark however that other mechanisms (e.g. the gravitational convection) could perhaps provide the dynamo energy.

The U and Th anti-neutrino fluxes and spectra

Uranium: $\Phi_{\bar{\nu}}/G = 3 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}$.

Cont. Spect. with $E_{\text{max}} = 3.26 \text{ MeV}$

Thorium: $\Phi_{\bar{\nu}}/G = 2.5 \cdot 10^5 \text{ cm}^{-2}\text{s}^{-1}$.

Cont. Spect. with $E_{\text{max}} = 2.25 \text{ MeV}$

Both can be detected by means of :

Remark that only Uranium contributes to events with:

$$E(e^+) > 0.45 \text{ MeV}$$

